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GLACIAL CANONS.

Historical Note.—This paper was presented before the American Association for the Advancement of Science at the Minneapolis meeting, where it was kindly read by Mr. Warren Upham in the absence of the author. A brief abstract was printed in the proceedings of that body for 1883, page 238. Subsequently, Dr. J. E. Hendricks, long editor of *The Analyst*, did the favor of reviewing the mathematical portions, and his suggestions are embodied in a note.

The paper is the fruit of field studies in the Sierra Nevada, mainly in the region about Lake Mono, and of subsequent office work in Salt Lake City, under the direction of I. C. Russell, then of the United States Geological Survey, in 1882 and 1883. The paper was not published because it was recognized that one of the most important phases of ice work (*i. e.*, the work at the bottom of the Bergschrund involved in the formation of cirques and rock basins) was not adequately treated. It was then, as it is now, the opinion of the author that ice work is concentrated and culminates in effectiveness in cirques, whether at the heads of water-carved tributaries (cyms or coombes) or in amphitheatres below ice-falls due to varigradational irregularities in the antecedent water-cut profiles, and that this concentration is proved and the correct analyses of the process suggested by the Bergsschrund in the one case and by seracs in the other; but the analysis is difficult, and neither then nor later have opportunities occurred for working it out. Recently this phase of ice work has been taken up by Mr. Willard D. Johnson, who brings to the work a rich fund of observation and an acute and vigorous mind, while at the same time the author finds the promise for the desired opportunity for further study fading away; so it is deemed best to publish in the present form, leaving extension and application to others. It may be observed that, while the treatment of the subject in this paper is analytic, the work was primarily synthetic and based directly on field observations and inferences in the magnificent field of the southern Sierra.

I.

Glacial cañons are characterized by several peculiar features: 1. They are **U** shaped rather than **V** shaped in cross-profile; 2. Small tributary gorges usually enter at levels considerably above the cañon-bottoms; 3. In longitudinal profile the cañon-bottoms are irregularly terraced—*i. e.*, made up of a series of rude steps of variable form and dimensions,—and some of the terraces are so deeply excavated as to form rock-basins occupied by lakelets; 4. The cañons are sometimes locally expanded into amphitheatres; 5. The cañon-bottom is not always obdurate rock, but may consist of coarse fragmental debris in which individual blocks are as deeply striated and as smoothly polished as are the most solid ledges, though they may rest so insecurely in their positions that a hand can overthrow them; and 6. The volume of glacial debris in moraine and valley deposits is but a small fraction of the cubic content of the cañon from which it was derived.

Of these features the first four suggest that glaciers are most effective engines of erosion, while the last two indicate that glacial erosion is inconsiderable. The source of the apparent discrepance may be sought through analysis of the agencies involved in the development of the four features first enumerated.

II.

Whatever be the physical cause of ice-flow, the motion of a glacier is unquestionably determined by (1) the weight of the ice, (2) the declivity of the channel, (3) the share of potential energy not expended in overcoming internal cohesion, and hence available in producing mass motion, and (4) the friction against bottom and sides of the channel; of which factors the last two (one of which is positive and the other negative) are indeterminate. The united effect of all—*i. e.*, the total sum of potential energy available in generating movement—may be denominated the *down-stream impulse* of the glacier. Such impulse, in combination with the simple *weight* of ice at any point, constitutes the *intensity* of glacial action at that point.

But, *ceteris paribus*, the measure of rock-grinding is the *friction* between the glacier and its bed. Now such friction is a complex function of the weight and down-stream impulse, and varies with, but probably less rapidly than, their product. The general law of friction, applicable under wide ranges of pressure and velocity, has never, indeed, been clearly formulated; and where the contiguous surfaces are so unlike as rock and ice the friction is scarcely known even in the simplest case.[1] In case of such substances, too, if detached rock-fragments intervene, they will project into the more yielding material and thereby increase the frictional surface; when the slip may either (1) occur in part on each side of the fragments (*i. e.*, the ice may flow over the fragments, while they themselves move at a slower rate over the valley-bottom, as has, indeed, been observed by Niles), or (2) may be confined to the inosculating rock-surfaces. Also, if a continuous sheet of comminuted debris intervene, the movement may be divided between its upper and lower surfaces; and if the intercalated sheet be thick, several planes of slip may exist within it and its own motion become differential. Again, if fragments of large angles and not greatly different diameters project into the ice or lie within a differentially-moving ground moraine, the unequal flow will most rapidly carry forward their summits, initiate rolling, and thus diminish friction (and at the same time, perhaps, produce "fluxion-structure"). It follows that the friction in any given case cannot be even approximately evaluated; and its expression must, therefore, include an indeterminate factor of considerable moment.

But, again, the disposition to attack the glacier-bed is

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[1] Tylor found that with a pressure of two pounds to the square inch the co-efficient of friction of ice upon ice was between 0.1 and 0.2, and concluded that glacier motion would be impossible without water to lubricate the bottom. Geol. Mag., Dec. II., Vol., II., 1875, p. 280.

(*ceteris paribus*) measured by the ratio between weight and down-stream impulse; for manifestly, if the weight be in excess, the predominant tendency must ever be to fix and retain in their places all bowlders, pebbles, sandgrains, and smaller particles; when the weight and impulse are as w and v in the diagram (fig. 1) their resultant will tend to retain rather than remove such fragments, and transportation will be limited to that due to friction and sub-glacial water; when the factors are equal, as are w' and v', their resultant will tend equally to retain and to remove particles, and the effects of friction and flowing water will be counteracted by the greater specific gravity of rock than ice; and when the ratio is as w'' to v'', the disposition will be to overturn and sweep forward all fragments. Also, the weight of ice tends to produce crushing of the rock in a degree probably increasing increasingly with its value. Finally, with increased weight will go increased pressure-liquefaction of the ice, and from this will result the antagonistic effects of reduced friction and augmented transportation. The last two agencies are variable, only very roughly determinate in the ordinary case, and generally of inconsiderable value. They may be thrown together as an unknown factor which, in conjunction with the predominant first agency, constitutes the *effectiveness* of glacial erosion at any point.

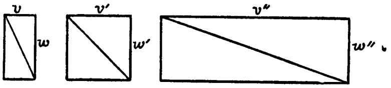

FIG 1.

The three elements of *intensity*, *friction*, and *effectiveness*, therefore, determine the rate of glacial erosion. To more succintly express their relations, let—

w=weight of ice at any point;
v=down-stream impluse at any point.
s=rock-surface in contact with any vertical prism of ice;
x=unknown factor in friction term; and
z=unknown factor in effectiveness term.

Then, denoting the three elements by their initials:

$$I=w \times v;$$

$$F=\frac{wvx}{s}\text{ ; and}$$

$$E=(1-z)\ \frac{v}{w}$$ [1]

Obviously, these elements are of unlike value in different parts of the cross-section of a glacial valley, and the rate of erosion is hence differential; but since important unknown factors are involved, no reliable expression either for the absolute rate of erosion at any point, or for the ultimate form of the glacial bed, can be directly deduced. The general tendency of glacial action may, however, be learned from separate consideration of the individual tendencies of the several agencies comprehended.

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[1] In the above statement, it has been the purpose to eliminate what is thought to be an element of uncertainty in the extension of the customary formula for friction to quantities so great and so peculiarly conditioned as those involved in the movements of great glaciers. It might be simpler also, as Dr. Hendricks points out, to reduce the determinants of glacier motion to those of positive action—viz., (1), the weight of the ice, (2), the declivity of the channel, and (3), the potential energy available in producing mass motion—by excluding the negative determinant, friction. The down-stream impulse might also be represented by $nw\ \sin\theta$, n being an unknown factor depending on molecular force, and hence involving temperature, etc. Then, making x the co-efficent of friction, the equations would become:

$$I=w \times nw\ \sin\theta;$$

$$F=w\ x\ \cos\theta;\text{ and}$$

$$E=(1-z) \times \frac{v+F}{w\ x\ \cos\theta}.$$

Or, introducing the factor $f\ (v)$ to represent the influence of velocity of flow in determining the friction; the last two equations would become:

$$\mathrm{F}=w\ x\ \cos\theta \times f\ (v);\text{ and}$$

$$\mathrm{E}=(1-z)\ \frac{v+F}{w\ x\ \cos\theta \times f\ (v)}.$$

It will be observed that this modification of the equations for *intensity*, *friction*, and *efficiency* do not materially affect the discussion, and do not in any way detract from the conclusions reached. The original equations are retained, however, in the opinion that they suggest, if they do not actually present, the more direct and serviceable mode of analysis.

It is a pleasure to acknowledge obligation to Dr. J. E. Hendricks, of Des Moines for working out the expressions in this note (January 25, 1885).

In such consideration let the ice be assumed to occupy a previously-formed gorge of the typical **V** form of water-cut cañons.

The weight of the ice varies directly with its thickness, and accordingly increases progressively from sides to center of the gorge. The tendency of this factor is hence to continually deepen the cañon and to perpetuate the **V** form.

Three of the four factors into which down-stream impulse may be resolved are of unequal value in different portions of the width of the glacier, and from such inequality the differential flow of ice-streams results; for from sides to center the weight increases uniformly, the available energy increases increasingly, and the friction probably increases less rapidly than the thickness; whence the impulse at the center must ever remain predominant. But if the ice-stream be conceived to consist of a parallel series of longitudinal vertical laminæ (for in the present discussion the vertical variation of flow is immaterial), it is evident that those at the edges will be retarded by the valley-sides, that the mediolateral laminæ will be equally retarded and accelerated by their unequally flowing neighbors, and that the central lamina will be retarded by the more slowly moving ice on either side; and if the mutual interaction of the various laminæ be considered, that the platted ordinates of flow will form a curved figure, and not a triangle homologous with the cross-section of the gorge (fig. 2). Such indeed is the case of differential ice-flow, as empirically established by Forbes, Agassiz, Tyndall, and other observers; though in the **V** gorge the curve would unquestionably be less flattened than in the **U** gorges within which the measured glaciers lie. On the whole, the disposition of the second factor must be to most energetically attack the valley-bottom, but at the same time to develop concavity of the valley-sides.

Summarizing, it appears that the general tendency of the intensity element is preëminently to deepen the cañon and slightly to transform the **V** to a **U** profile.

Of the factors peculiar to friction, that of indeterminate value doubtless suffers increasing relative diminution as the depth of ice increases, and its platted ordinates (expressed in terms of the valley-profile) will hence form a curve of materially less depth than the triangle formed by the tangents to its extremities (fig. 2). The disposition of the factor is accordingly to widen the gorge and develop the **U** profile.

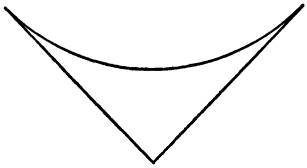

Fig 2.

With the less lateral velocity common to ice-streams will go reduced lateral friction, and hence erosion, in a ratio corresponding to the velocity curve; and for a second reason, therefore, will concavity of the valley-sides be engendered and developed; though the concurrent disposition will be to deepen the gorge.

Whenever concavity of the valley-sides obtains, the contact surface of the vertical prism will become variable. If, now, friction vary approximately with the pressure of the incumbent ice, the consequent erosion will diminish with the increasing slope toward the edges of the glacier; when the disposition will be to deepen the gorge and restore the **V** form; but if the friction vary more nearly with the contact-area, it will increase with the slope, and the resulting erosion will tend to widen the gorge and, in another manner, to restore the **V** profile. Whichever tendency obtains will, however, be secondary and ever subordinate to that of the principal factors of friction. (Subglacial water will at once reduce friction and promote transportation directly and corrasion indirectly; also it will tend, *ceteris paribus*, to form a continuous film between ice and rock reaching upward to 0.92 of the thickness of the glacier, or, if the glacial surface be highly convex, perhaps quite to its margins. On the whole, then, its influence in any direction must be slight, and its effect may be disregarded).

Combining the several antagonistic factors, it appears uncertain whether the general tendency of the friction element is to widen or deepen the gorge, but certain that it is to develop concavity of the valley sides and the **U** form of cañon.

Since the third and fourth factors in down-stream impulse (available potential energy and friction) are indeterminate, the problem as to the declivity required to render such impulse equal to the weight at any point in a given glacier, or even as to whether such equality ever obtains in nature, cannot be analytically solved; and very few observations showing the relative value of these components have ever been made. Niles,[1] however, found that in the Great Aletsch glacier the ice usually rides upon projecting rugosities and seldom fills the intervening depressions of its bed, and that a bowlder (itself slowly moving) three feet high had formed an inverted trough thirty feet long in the base of the incumbent ice; whence the down-stream impulse must have exceeded ten times the weight. Bonney,[2] also, in the Glacier des Bois and the Glacier d' Argentiere, found all broad and gentle depressions in the glacier beds filled with impressed ice, the narrower depressions not quite filled, the lee of projecting knobs protected for a distance equal to their height, and bowlders lying *in situ* beyond the present terminus of the ice glaciated above and below (showing that here also motion took place along the two planes), all of which phenomena indicate that, in these glaciers, the down-stream impulse is in excess of weight, but in a less degree than in the Great Aletsch. The several observations then demonstrate (1), that down-stream impulse may greatly exceed weight, and (2), that the relation is variable. All were in the upper portions of the valleys where the declivity is great (15° to 20° in the examples described by Bonney), and where the office of the glaciers is preëminently one of erosion. Now ordinary valleys, whether occupied by streams or glaciers, are of progressively diminishing declivity from source to terminus; ordinary glacial valleys exhibit successive zones of active erosion, feeble erosion, slight deposition, and abundant deposition in passing from their upper reaches to the broader valleys into which they embouch or upon the plains with which they merge; and in such cases the down-stream impulse must wane to practically nothing at the extremities of the glaciers, and must hence greatly fail of the weight. It follows that at some point (or at diverse points) in every extended glacier-course the components weight and impulse are equal at the centre of the glacier.

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[1] Proc. Boston Soc. Nat. Hist., XIX., 1878, 330; Am. Jour. Sci., XVI., 1878, 366.

[2] Geol. Mag., Dec. II., Vol. III., 1876, 197.

Since glacier ice but slightly approaches perfect fluidity and the flow of the center is greatly retarded by the sides, the ratio of impulse to weight (and with it the effectiveness) continually and largely increases from center to sides: if the central effectiveness be just zero, that at the sides will nevertheless remain important; if it be minus centrally, it may still be considerable laterally; and however great may be its value at the center, it must have far greater value at the sides. The disposition, then, will ever be to protect the bottom and equally to attack the sides of the valley; and since the down-stream impulse of the several parallel laminæ forms a curve when platted, so will the disposition also be to form concave valley-sides.

Of the unknown factor in the effectiveness term, the first component (rock-crushing) can be but trivial in the ordinary case, while the second (pressure-liquefaction) exercises antagonistic influences. It may, accordingly, be safely neglected.

Collectively, the tendencies of the third element of glacial erosion are (1) to effectually protect the valley-bottom throughout a considerable portion of the glacier course, (2) to develop the **U** form of canon, and (3) to materially increase the relative width of the gorge.

The fifth feature of glacial cañons is explained by the operation of this element, and in turn establishes the importance of the element.

Recapitulating, it appears that of the several elements involved in glacial erosion, the first tends to deepen the gorge and slightly to develop the **U** form, the second to develop the **U** form, and perhaps very slightly to deepen the gorge, while the third and predominant one tends strongly to widen the gorge and protect its bottom, and less strongly to develop the **U** form. It follows that the general tendency of glaciers must be to widen rather than deepen the valleys they occupy, and to transform **V** to **U** cañons. Also, since the typical **U** gorge is just such as would result from temporary occupancy of a **V** gorge by a glacier, while the ordinary ratio of width to depth is less than would obtain were the gorge eroded by glacial action exclusively, it follows again that the characteristic glacial cañons must be only modified stream-cañons.

This conclusion explains, and is equally and directly corroborated by, the first and sixth features of glacial cañons. It also fully warrants the assumption, in the following as in the foregoing discussion, of originally **V** shaped glacier-beds.

III.

As elsewhere shown,[1] corrasion of a stream is a function of its volume, and, *ceteris paribus*, varies with, but less rapidly than that element. In a region of rapid corrasion then, the main stream must (unless the declivity be materially unlike) more rapidly corrade its channel than does its minor tributary; and the tributary cañon must accordingly enter its principal over a rapid or at least a convex curve in longitudinal profile.

If now the main cañon become filled with ice and be transformed from the **V** to the **U** type by its action, the distal extremity of the tributary will be cut off and the original stream-formed declivity replaced by the precipitous side-wall of the normal glacier valley (fig. 3); and this result will follow whether the tributary be filled with or free from ice, provided corrasion at the cañon-mouth be not relatively increased in a considerable degree.

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[1] "The Formation of River Terraces" (recently published in Eleventh Annual Report U. S. Geological Survey, 1891, pp. 259–272).

It follows that the second feature of the typical glacial cañons may naturally result from temporary occupation of water-cut cañons by ice, and that it does not necessarily argue profound glacial erosion.

IV.

In obedience to the law of varigradation,[1] all and particularly smaller streams tend to depart in a minor degree from uniform gradient, and to develop in their channels a longitudinal profile

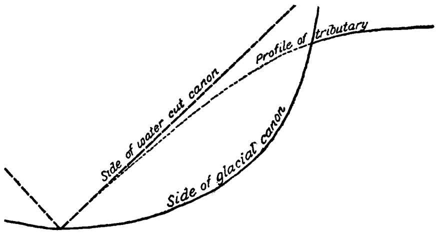

FIG. 3.

of slightly variable declivity; this law finding expression in the alternating pools and rapids of mountain brooks and in the always perceptible and often conspicuous alternations of greater and less declivity in the courses of water-cut cañons.

If now an otherwise uniform **V** cañon of irregular gradient become occupied by a glacier, the flow, varying as it does with the declivity, will become unequal and the ice will tend to accumulate on the planes of low declivity until it approaches a uniform surface slope; when the weight of ice at different points in the medial or other longitudinal plane of the glacier will become variable, and will reach a maximum over the greatest depression (fig. 4). With such increased weight will go (*a*) direct increase of intensity with the augmentation of its principal factor, (*b*) indirect increase of intensity in virtue of the office of weight as a function of the down-stream impulse, and (*c*) direct diminution of intensity in consequence of the absolutely reduced down-stream impulse; also (*d*) material increase of friction with the augmentation of its principal factor, and (*e*) less material diminution of friction in consequence of the reduced impulse; and finally (*f*), direct diminution of effectiveness with the absolute decrease of impulse, (*g*) indirect diminution of effectiveness in consequence of the relative decrease of the same factor, and (*h*) direct but slight increase of effectiveness in virtue of the operation of the obscure factor of rock-crushing and pressure-liquefaction; or, summarily, increase in intensity, slight increase in friction, and decrease in effectiveness.

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[1] Op. cit, p. 295.

Now, in view of the obscure and antagonistic though interdependent relations involved, it is evident that without exhaustive quantitative investigation (impossible in the present absence of knowledge concerning friction between ice and other substances) it cannot be determined in the ordinary case whether the disposition will be to erode the more rapidly where weight increases at the expense of declivity, or where the reverse occurs; but it appears quite certain that where the surface declivity materially exceeds that at the base, and where, accordingly, the impulse is not reduced proportionally to the declivity of the channel, erosion must progressively increase with the weight. If so, the tendency of glaciers must be to cumulatively intensify the irregularities in gradient normal to water-cut cañons.

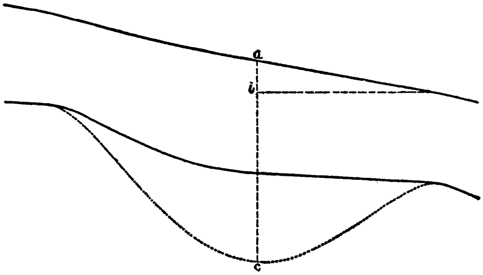

Fig. 4.

But corrasion and transportation in any part of a glacier-bed are limited directly by flow of ice and indirectly by coincident flow of subglacial water. Now, loss of effectiveness through absolute and relative increase of weight must eventually become potent in retarding direct excavation of the depression; also, whenever the depression becomes so considerable as to possess reverse slope toward its distal extremity, gravity will no longer enhance, but instead oppose, direct transportation of detritus; again, with increased depth of depression will go increased cross-section and concomitant and material diminution of velocity and eroding capacity in the ice-stream; and finally, the longitudinal perimeter of the depression must continually increase until the fricton along it approaches and ultimately equals the shearing strength of the ice along its chord, whence the movement of the basal segment must concurrently diminish and gradually cease. In like manner, when the normal slope becomes reversed, gravity will oppose and not enhance transportation by subglacial water; also, as the reverse slope increases, the flow of such water will become sluggish and its capacity diminished; and finally, when the depth of depression below its distal rim reaches 0.92 of the maximum depth of ice (or when *b-c* equals 0.92 *a-c*, fig. 4), the subglacial water will assume static equilibrium, the incumbent ice will suffer flotation, and both corrasion and transportation will practically cease. Thus the excavation of depressions by direct ice-action has a definite though indeterminate limit, and can probably never exceed a moderate fraction of the depth of the ice; and thus also indirect glacial erosion in depressions through the coöperation of subglacial water alike in corrasion and transportation will remain effective until the depth of excavation approaches the thickness of the incumbent ice; whence, in the general case, the measure of maximum excavation of rock-basins is a large fraction of the depth of the glacier.

(Evidently embouchures of valleys, zones of abrupt diminution in declivity, points at which for any reason glaciers terminate for considerable periods, broad cross-valleys beneath continuous ice-sheets, and all localities where the surface slope of the ice materially exceeds the slope of its base, will form as definite loci of active excavation as do the ordinary planes of low declivity developed by varigradation; and at such localities, accordingly, glacial lakes, the submerged rock-basins characteristic of fjords, and other evidences of energetic ice-action remain after the melting of the ice.)

It follows, then, that the third feature of glacial cañons may result simply from glacial occupation of water-cut cañons; and since in the common mountain region from which the glaciers have completely disappeared the irregularities of gradient peculiar to such cañons are not greatly intensified, while glaciated rock-basins are comparatively rare and of slight depth, it equally follows that the occupation was only temporary, and the sum of glacial erosion relatively inconsiderable.

V.

The immediate effect of the origin of a tributary cañon in a developing drainage-system is the exposure of a greater length of canon-wall to degradation; from which effect in turn results (under certain conditions of homogeneity of terrane and uniformity of altitude in the region, and hence of repeated bifurcation and wide dispersal of the branches of the nascent tributary) the formation of an amphitheatre opening into the main cañon. Then, after the considerable development of the tributary, its disposition will be, as shown by Warren,[1] to dam the main stream and diminish the declivity above its confluence; whereby lateral corrasion will increase at the expense of vertical corrasion there. Thus, by increased lateral corrasion the amphitheatre will ever tend to expand within certain limits immaterial in this discussion. Such amphitheatres, exhibiting the tortuous outlines characteristic of fluvial erosion, have been well illustrated by Dutton,[2] and are common features in many mountain regions.

If now a glacier enter and fill such an amphitheatre, its rate of flow and similarly its rate of erosion on the given area will be reduced by increase of width and depth; though if (as is probable) erosion varies more nearly with the weight than the velocity, its amount will increase absolutely, and the expanded valley will tend in a stronger degree than that measured by the ratio of the inverse volumes to assume the general form characteristic of contracted glacial gorges. As in the contracted gorge, too, lateral effectiveness will remain predominant; but the effective energy of the glacier will be mainly concentrated upon the obstructive angles, spurs, and cusps of the irregular water-carved walls, and the removal of these and the rounding out of the amphitheatre will be in the first work of the glacier. Again, the partial rigidity of the ice-mass will lead to culmination of pressure about the distal extremity of the amphitheatre, and to consequent extension of its boundaries beyond the confluence of the tributary by which its water-fashioned prototype was originated.

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[1] "An Essay Concerning Important Physical Features Exhibited in the Valley of the Minnesota River," 1874–7; and elsewhere.

[2] "Tertiary History of the Grand Cañon District," 1882, Chapter IX, and maps accompanying.

It follows that glaciated amphitheatres may be merely water-carved valley expansions modified by temporary ice-action into regularity of contour (as are, for instance, those of the Faeröe Islands[1]), and that they do not necessarily argue profound glacial erosion.

VI.

Summarizing the chief effects of the several agencies involved in the development or the characteristic features of glacial cañons, it appears that temporary occupancy of a typical water-cut cañon by glacier ice will (1) increase the width, (2) change the **V** to a **U** cross-profile, (3) cut off the terminal portions of tributary cañons, and thus relatively elevate their embouchures, (4) intensify certain irregularities of gradient in the cañon-bottom, (5) excavate rock-basins, (6) develop amphitheatres, and, in general, transform such cañon into an equally typical glacial cañon. It follows that these features do not necessarily imply extensive glacial excavation or indicate that glaciers are superlatively energetic engines of erosion.

W J McGee.

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[1] J. Geikie, "Geology of the Faeröe Islands," Trans. Roy. Soc., Edin., 1882.